An Asymptotic Parallel-in-Time Method for Highly Oscillatory PDEs

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Since the beginning of numerical weather prediction it has been understood that mathematical theories can lead not only to deeper understanding but also improve numerical representation of the dynamics in atmosphere and ocean models. The difficulties inherent in computational models have lead to, among other subjects, Charney's derivation of QG, theoretical investigations of the 'slow manifold' and later, nonlinear normal mode initialisation for climate models. In this talk the saga unfolds into the era of exascale computing by using mathematical theories developed for oscillatory partial differential equations to develop new numerical methods suitable to today's heterogeneous computing architectures. I will give this talk in three parts:

1. I will briefly explain how exascale computers are different than parallel computers, why the computer architecture community is going this direction, and what this means for the future of computing for atmosphere and ocean models.

2. I will sketch the thinking behind a numerical discovery where Terry Haut and I transformed a mathematical method used for understanding fast singular limits of PDES into a time-stepping numerical method. This new algorithm has enabled a type of computing for systems of equations with oscillatory stiffness called asymptotic-parallel-in-time. Parallel-in-time methods have been known since an influential paper by Lions, Maday, and Turinici in 2001 but until now had most of their success for dissipative systems, not oscillatory systems that we face in the atmosphere and ocean. The 'slowish' dynamics (zero frequency plus near-resonances) that underpins the new algorithm allows us to take time steps far larger than that dictated by the fast waves and gives us a parallel speed up of 100 over conventional methods for the shallow water equations.

3. Last, if there is time, I will show selected results of work with Jared Whitehead where, looking through the lens of 'slow' dynamics in 3 distinguished limits of the 3D Boussinesq equations we watch how white noise forcing moves through a system partitioned into slow and non-slow dynamics and find that the non-slow dynamics serves to move energy onto and off of the 'slow manifold' -- changing the evolution of the 'slow' dynamics. The degree to which this happens depends on the system and how close the dynamics is to the singular limit.