Sparse Adaptive Finite Elements for Radiative Transfer

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The linear radiative transfer equation, a partial differential equation for the radiation intensity $u(\mathbf{x}, \mathbf{s})$, with independent variables $\mathbf{x} \in D \subset \mathbb{R}^n$ in the physical domain D of dimension $n = 2, 3$, and angular variable $\mathbf{s} \in S^2 := \{\mathbf{y} \in \mathbb{R}^3 : |\mathbf{y}| = 1\}$, is solved in the $n + 2$-dimensional computational domain $D \times S^2$. We propose an adaptive multilevel Galerkin FEM for its numerical solution. Our approach is based

- on a stabilized variational formulation of the transport operator,
- on so-called sparse tensor products of two hierarchic families of Finite Element spaces in $H^1(D)$ and in $L^2(S^2)$, respectively,
- on the local use of full tensor product spaces for the resolution of influx boundary conditions,
- on thresholding techiques to adapt the discretization to the underlying problem,
- on subspace correction preconditioning of the resulting large linear system of equations.

An a-priori error analysis shows, under strong regularity assumptions on the solution, that the sparse tensor product method is clearly superior to a discrete ordinates method, as it converges with essentially optimal asymptotic rates while its complexity grows essentially only as that for a linear transport problem in $\mathbb{R}^n$. Numerical experiments for $n = 2$ on a set of example problems agree with the convergence and complexity analysis of the method and show that introducing adaptivity can improve performance in terms of accuracy vs. number of degrees even further.

References

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