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Exercise 2 for the lecture

# Numerics III

SoSe 2012

**Due: till Thursday, May 3rd, 2012, 12 o'clock**

**Problem 1** (4 TP)
Classify the following pdes.

a) $-\mathrm{div}(\alpha(x)\nabla u) = 0, \qquad \alpha \in C^1(\Omega), \quad \alpha(x) \geq \alpha_0 > 0.$

b) $\varepsilon \Delta u - \vec{\beta}\nabla u = 0, \qquad$ (i) for $\varepsilon \neq 0$, (ii) for $\varepsilon = 0$.

c) $(c_0^2 - u_x^2)u_{xx} - 2u_x u_y u_{xy} + (c_0^2 - u_y^2)u_{yy} = 0 \qquad c_0 > 0.$

**Problem 2** (4 TP)
Consider the Cauchy problem

$$\begin{aligned} a u_{xx} + 2b u_{xy} + c u_{yy} &= d && \text{in } \mathrm{R}^2, \\ u &= u_0 && \text{on } \gamma, \\ \frac{\partial}{\partial n} u &= u_1 && \text{on } \gamma. \end{aligned}$$

with a smooth curve $\gamma : I \to \mathrm{R}^2$ in $\mathrm{R}^2$. Show that the condition

$$\det(s) = a\gamma_2'(s)^2 - 2b\gamma_1'(s)\gamma_2'(s) + c\gamma_1'(s)^2 \neq 0$$

is sufficient to compute $u_{xxx}$, $u_{xxy}$, $u_{yyx}$ and $u_{yyy}$ in $\gamma(s)$.

**Problem 3** (3 TP)
Derive d'Alembert's solution of the following initial value problem for the wave equation

$$\begin{aligned} u_{xx} - u_{yy} &= 0 && \text{for } (x,y) \in \mathrm{R} \times \mathrm{R}_+ \\ u(x,0) &= u_0(x) && \text{for } x \in \mathrm{R}, \\ u_y(x,0) &= u_1(x) && \text{for } x \in \mathrm{R}. \end{aligned}$$

**Problem 4** (4 TP)
Consider the initial value problem

$$\rho_t + v_{\max}(\rho(1 - \rho/\rho_{\max}))_x = 0, \quad t > 0, \qquad \rho(x, 0) = \rho_0(x) \tag{1}$$

for the nonlinear car conservation law with $v_{\max}$ and $\rho_{\max}$ describing maximal velocity and density, respectively.

a) Compute the characteristics of (1).

b) Show by a counterexample that smoothness of the initial data $\rho_0$ does not imply existence of a classical solution $\rho \in C^1(\mathrm{R} \times \overline{\mathrm{R}}_+)$.