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Exercise 6 for the lecture

Numerics III

SoSe 2012

Due: till Thursday, May 31th, 2012, 12 o'clock

Problem 1 (4 TP)
Consider the space

$$\mathcal{L}(V, W) = \{L : V \to W | L \text{ linear and bounded}\}.$$

Show that $\mathcal{L}(V, W)$ is a Banach space, if W is a Banach space.

Problem 2 (2 TP)
Let us consider the inner product space

$$H_C = \{v \in C^1(\Omega) \cap C(\bar{\Omega}) \,|\, v|_{\partial\Omega} = 0, \|v\|_{H^1(\Omega)} < \infty\},$$

equipped with the inner product $(\cdot, \cdot)_{H^1(\Omega)}$. Find a bilinear form on H_C, which is positive definite, but not H_C-elliptic.

Problem 3 (2 TP)
Let H be a Hilbert space with scalar product $(\cdot, \cdot)$ and $S \subset H$ a closed subspace. Show that

$$Pu = \underset{v \in S}{\operatorname{argmin}} \left(\frac{1}{2}(v, v) - (u, v) \right)$$

defines an orthogonal projection $P : H \to S$, the so-called *Ritz projection.*

Problem 4 (4 TP)
Let Ω be a bounded domain and $u, v \in H^1(\Omega)$. Show that the distributional derivative of uv satisfies the product rule

$$\partial_i(uv) = (\partial_i u)v + u(\partial_i v) \ .$$

Is uv always in $H^1(\Omega)$? Give a proof or counter-example.

Problem 5 (4 TP)
Which of the following functions is an element of $H^1((-1,1))$ (and why or why not, respectively)?

a) $x \mapsto \sqrt{x+1}$

b) $x \mapsto |x|$

c) $x \mapsto |x| + \chi_{\mathbb{Q}}(x)$

Here, $\chi_{\mathbb{Q}}$ is the characteristic function of the set $\mathbb{Q}$, i.e., it is 1 on $\mathbb{Q}$ and 0 otherwise.