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Exercise 7 for the lecture

Numerics III

SoSe 2012

Due: till Thursday, June 7th, 2012, 12 o'clock

Problem 1 (4 TP)
Consider the space $H^{\frac{1}{2}}(\partial\Omega) := \mathrm{tr}(H^1(\Omega))$.

a) Define a norm on $H^{\frac{1}{2}}(\partial\Omega)$, such that $\mathrm{tr} : H^1(\Omega)/\ker(\mathrm{tr}) \to H^{\frac{1}{2}}(\partial\Omega)$ is an isometry with respect to the quotient norm.

b) Show that $c\|v\|_{L^2(\partial\Omega)} \leq \|v\|_{H^{\frac{1}{2}}(\partial\Omega)} \ \forall v \in H^{\frac{1}{2}}(\partial\Omega)$.

Problem 2 (4 TP)
Let $(\Omega_n)_{n\in\mathbb{N}}$ with $\Omega_n = \{x \in [0,1] |\ x = i/n, i = 0, \dots, n\}$ be a sequence of grids and $(v_n)_{n\in\mathbb{N}}$ with $v_n \in \{v \in C[0,1] |\ v|_{[(i-1)/n,\ i/n]} \in \mathcal{P}^1, i = 1, \dots, n\}$ a sequence of functions with the properties

$$\max_{i=1,\dots,n} |v_n|_{[(i-1)/n,\ i/n]}| \leq c_1 \quad \text{and} \quad \max_{i=1,\dots,n} |n(v_n(i/n) - v_n((i-1)/n))| \leq c_2 \quad \forall n \in \mathbb{N}.$$

Show that there exists a subsequence (v_{n_j}) and a function $v \in C[0,1]$, such that

$$v_{n_j} \to v,\ j \to \infty \quad \text{in } L^2.$$