
ERGODIC THEORY AND TRANSFER OPERATORS
— QUESTION SHEET 10 —

Summer 2015

Exercise 1 (Monte Carlo estimation of the Ulam matrix)

Let \hat{P}_n be the numerical approximation by Monte Carlo sampling of the Ulam matrix P_n from the lectures. Show that \hat{P}_n inherits positivity and integral preservingness from P_n ; that is \hat{P}_n is also a stochastic matrix.

Exercise 2 (Ulam's method in 1d)

Implement Ulam's method in 1d for the interval $I = [0, 1]$, by utilizing Monte Carlo sampling to approximate the Ulam matrix P_n , $n = 50, 100, 200$. Use it to compute approximate invariant densities for the transformations

$$T_1(x) = 2x \pmod{1}, \quad T_2(x) = 4x(1-x), \quad T_3(x) = \begin{cases} \frac{x}{1-x}, & 0 \leq x < \frac{1}{2} \\ 2x-1, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Do the results reflect your expectations? Note that T_3 doesn't have an ACIM (cf. 3.10 from the lectures, and Sheet 8, Exercise 4).