Ergodic Theory and Transfer Operators
— Question sheet 12 —

Summer 2015

Exercise 1 (Rokhlin metric)

Show that the Rokhlin metric $d(\xi, \eta) = H(\xi|\eta) + H(\eta|\xi)$ is a metric for finite partitions; i.e. that for finite partitions ξ, η, and ζ holds

(a) $d(\xi, \eta) \geq 0$ with equality only if $\xi = \eta$, and

(b) $d(\xi, \eta) \leq d(\xi, \zeta) + d(\zeta, \eta)$.

Exercise 2 (Entropy of Bernoulli shifts)

Let $p \in \mathbb{R}^k$ be a probability vector. Show that the p-Bernoulli shift has entropy $-\sum_{i=1}^k p_i \log p_i$.

Exercise 3 (Entropy of products)

Show that the entropy of the product $T_1 \times T_2$ of two probability preserving transformations T_1 and T_2 is the sum of the entropies of T_1 and T_2.

Remark: Recall that we introduced the product as a model for independent dynamical systems (§1.21), and entropy has the additivity property $H(\xi \vee \eta) = H(\xi) + H(\eta)$ exactly if ξ and η are independent partitions (Proposition 4.3 (d)). Hence, the result in this exercise reflects our intuition how entropy should behave for independent objects.