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Exercise 1 for the lecture

Numerics III

SoSe 2015

Due: till Tuesday, 28. April

Problem 1

Let $\Omega \in \mathrm{R}^d$ with a sufficiently smooth boundary $\partial\Omega$, e.g. with a continuously differentiable parametrization, and $v, w \in C^1(\overline{\Omega})$. Show that

$$\int_\Omega (v_{x_i} w + v w_{x_i})\, dx = \int_{\partial\Omega} v w n_i \, d\sigma, \qquad i = 1, \ldots, d,$$

where $n = (n_1, \ldots, n_d)$ is the outward-oriented normal on $\partial\Omega$.

Problem 2

Let $v, w \in C^2(\overline{\Omega})$ and Ω as above. Use your results from problem 1 to prove Green's identities:

a) $\int_\Omega (v\Delta w + \nabla v \nabla w)\, dx = \int_{\partial\Omega} v \frac{\partial w}{\partial n}\, d\sigma$

b) $\int_\Omega (v\Delta w - w\Delta v)\, dx = \int_{\partial\Omega} \left(v \frac{\partial w}{\partial n} - w \frac{\partial v}{\partial n}\right)\, d\sigma$