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Exercise 2 for the lecture

# Numerics III

SoSe 2015

**Due: till Tuesday, 5. May**

**Problem 1**

Let $\Omega \subset \mathrm{R}^2$ be a bounded domain with sufficiently smooth boundary and

$$H_C = \{v \in C^1(\overline{\Omega}) \mid v|_{\partial\Omega} = 0\}.$$

Prove that the variational equaltity

$$u \in H_C : \qquad \int_\Omega \nabla u \cdot \nabla v \, dx = \int_\Omega f v \, dx \qquad \forall v \in H_C$$

has at most one solution.

**Problem 2**

Consider a domain $\Omega \subset \mathrm{R}^2$ with a disjoint decomposition $\partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_R$ of the boundary for Dirichlet, Neumann and Robin conditions for the boundary value problem

$$\begin{aligned} -\Delta u(x) &= f(x) && \forall x \in \Omega \\ u(x) &= 0 && \forall x \in \Gamma_D \\ \frac{\partial u}{\partial n}(x) &= g_N(x) && \forall x \in \Gamma_N \\ u(x) + \beta \frac{\partial u}{\partial n}(x) &= g_R(x) && \forall x \in \Gamma_R \end{aligned}$$

with $f \in C(\overline{\Omega})$ and $\beta > 0$ where $n$ is the unit outer normal vector on $\partial\Omega$. Derive a variational equality

$$u \in V \qquad a(u, v) = l(v) \qquad \forall v \in V$$

for this boundary value problem and specify the space $V$, the bilinear form $a(\cdot, \cdot)$ and the functional $l$.