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Exercise 3 for the lecture

Numerics III

SoSe 2015

Due: till Tuesday, 12. May

Problem 1
Classify the following pdes.

a) $-\mathrm{div}(\alpha(x)\nabla u) = 0, \qquad \alpha \in C^1(\Omega), \quad \alpha(x) \geq \alpha_0 > 0.$

b) $\varepsilon \Delta u - \vec{\beta}\nabla u = 0, \qquad$ (i) for $\varepsilon \neq 0$, (ii) for $\varepsilon = 0$.

c) $(c_0^2 - u_x^2)u_{xx} - 2u_x u_y u_{xy} + (c_0^2 - u_y^2)u_{yy} = 0 \qquad c_0 > 0.$

Problem 2
Consider the Cauchy problem

$$\begin{aligned} au_{xx} + 2bu_{xy} + cu_{yy} &= d && \text{in } \mathrm{R}^2, \\ u &= u_0 && \text{on } \gamma, \\ \frac{\partial}{\partial n} u &= u_1 && \text{on } \gamma. \end{aligned}$$

with a smooth curve $\gamma : I \to \mathrm{R}^2$ in R^2. Show that the condition

$$\det(s) = a\gamma_2'(s)^2 - 2b\gamma_1'(s)\gamma_2'(s) + c\gamma_1'(s)^2 \neq 0$$

is sufficient to compute u_{xxx}, u_{xxy}, u_{yyx} and u_{yyy} in $\gamma(s)$.

Problem 3
Derive d'Alembert's solution of the following initial value problem for the wave equation

$$\begin{aligned} u_{xx} - u_{yy} &= 0 && \text{for } (x,y) \in \mathrm{R} \times \mathrm{R}_+ \\ u(x,0) &= u_0(x) && \text{for } x \in \mathrm{R}, \\ u_y(x,0) &= u_1(x) && \text{for } x \in \mathrm{R}. \end{aligned}$$