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Exercise 6 for the lecture

Numerics III

SoSe 2015

Due: till Tuesday, 2. June

Problem 1

Consider the space $X = C^1([0,1])$, equipped with the norms

$$\|v\|_\infty = \sup_{x\in[0,1]} |v(x)|, \qquad \|v\|_{\infty,1} = \|v\|_\infty + \|v'\|_\infty.$$

Show that the identity $Id : (X, \|\cdot\|_\infty) \to (X, \|\cdot\|_{\infty,1})$ is not continuous and conclude that norms are in general not equivalent in infinite dimensional spaces.

Problem 2

Consider the space

$$\mathcal{L}(V,W) = \{L : V \to W | L \text{ linear and bounded}\}.$$

Show that $\mathcal{L}(V,W)$ is a Banach space, if W is a Banach space.

Problem 3

Let us consider the inner product space

$$H_C = \{v \in C^1(\Omega) \cap C(\bar{\Omega}) \,|\, v|_{\partial\Omega} = 0, \|v\|_{H^1(\Omega)} < \infty\},$$

equipped with the inner product $(\cdot,\cdot)_{H^1(\Omega)}$. Find a bilinear form on H_C, which is positive definite, but not H_C-elliptic.

Problem 4

Let H be a Hilbert space with scalar product $(\cdot,\cdot)$ and $S \subset H$ a closed subspace. Show that

$$Pu = \operatorname*{argmin}_{v\in S} \left(\frac{1}{2}(v,v) - (u,v)\right)$$

defines an orthogonal projection $P : H \to S$, the so-called *Ritz projection*.