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Exercise 8 for the lecture

Numerics III

SoSe 2015

Due: till Tuesday, 16. June

Problem 1
Let $\Omega \in \mathrm{R}^2$ be a domain with polygonal boundary and $\mathcal{T}$ a triangulation of Ω. Show that

$$S^{(m)} \subset H^1(\Omega),$$

and derive the weak derivative of a function $v \in S^{(m)}$.

Problem 2
Consider the grid $a = x_0 < x_1 < \ldots < x_n = b$ on the interval (a, b). Let u be the solution of

$$u \in H_0^1(a,b) : \qquad (u', v') = (f, v) \qquad \forall v \in H_0^1(a,b)$$

and $u_h \in S_h$ the approximation in the linear finite element space $S_h \subset H_0^1(a,b)$ on the above grid. Show that u_h coincides with the linear interpolation of u on the grid, i.e. $u_h(x_i) = u(x_i)$ for $i = 0, \ldots, n$.

Problem 3 (6 programming points)
To solve a variational problem in a finite dimensional space V we need to assemble a matrix $A \in \mathrm{R}^{n \times n}$ with $A_{i,j} = a(\lambda_i, \lambda_j)$ where $\{\lambda_1, \ldots, \lambda_n\}$ is a basis of V.

a) Make yourself familiar with the Matlab programms `basis.m` and `quadrature.m` on the homepage.

b) Write a Matlab programm `A = assemble_stiff(S, B, Q)`, which assembles the bilinearform

$$\int_{\mathcal{T}} \nabla u(x) \cdot \nabla v(x) dx$$

for the basis $\{\lambda_{i,\tau}\}$ on a triangle τ given by the columns of the matrix `S`. The basis $\{\lambda_{i,\tau}\}$ on τ results from a transformation of the basis given by `B` on the unit simplex. Use the quadrature rule `Q` for evalution of the integrals.
Test your programm with `B = basis(1)`, `Q = quadrature(1)` and three triangles of your choice. What happens, if you scale one triangle with different factors h?

c) Analog to b) write a MATLAB programm `M = assemble_mass(S, B, Q)`, which assembles the bilinearform

$$\int_\tau u(x)v(x)dx.$$

Add a Gauß quadratur rule to `quadrature.m`, which is of order $p \geq 2$ on the unit simplex in R^2. Test your programm with `B = basis(1)` and the new quadratur rule `Q = quadrature(2)` and triangles of your choice. What happens, if you scale one triangle with different factors h?