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7th exercise for the lecture

Numerics III

Summer Term 2016

http://numerik.mi.fu-berlin.de/wiki/SS_2016/NumericsIII.php

Due: Wednesday, June 22nd, 2016 (10:15 a.m.)

Exercise 1 (4 TP)
Let $\Omega = B_1(0) \subseteq \mathbb{R}^2$. Show that the map u defined by $u(x) = \log\log\left(1 + \|x\|^{-1}\right)$ belongs to $H^1(\Omega)$.

Exercise 2 (4 TP)
Let $\Omega = B_1(0) \subseteq \mathbb{R}^n$. Show that there can not exist a bounded linear operator

$$T\colon L^2(\Omega) \to L^2(\partial\Omega)$$

such that $Tu = u|_{\partial\Omega}$ for all $u \in C(\overline{\Omega}) \cap L^2(\Omega)$.

Exercise 3 (4 Extra TP)
Let $\Omega = B_1(0) \subseteq \mathbb{R}^n$. Show that $H^1(\Omega)$ is not complete in $L^2(\Omega)$ with respect to the $L^2(\Omega)$-norm.

Exercise 4 (4 Extra TP)
Let $\Omega = (0,1)^2$ and $Q := \mathbb{Q} \cap \Omega$. Construct a function $u \in H^1(\Omega)$ such that u has a singularity in each point $q \in Q$.

Please turn over...

Exercise 5 (8 PP)
Suppose a triangulation T_i, $i \in 1, \ldots, m$ of some domain $\Omega = \bigcup_{i=1}^m T_i \subseteq \mathbb{R}^2$.

a) Read the Matlab documentation of the function `initmesh` and make yourself familiar with the represention of a triangulation in Matlab using a point matrix `p`, an edge matrix `e` and a triangle matrix `t`.

b) Write a function

```
[nodes, weights] = TriangleQuadratureFormula( n )
```

that returns the nodes and weights for a quadrature formula of order n over the reference triangle $\text{co}\{(0,0),(0,1),(1,0)\} \subset \mathbb{R}^2$. You may restrict yourself to $n \in \{1, \ldots, 4\}$. Appropriate quadrature nodes and weights can be found, e. g., in *Approximate calculation of multiple integrals* by A.H. Stroud.

c) Create another function

```
v = LocalTriangleQuadrature( f, x, n )
```

where f is a function handle, the matrix $x \in \mathbb{R}^{2\times 3}$ contains the vertices of a triangle as column vectors, and n is the quadrature order parameter. The result v should be the approximate integral of f over the triangle using the quadrature formula with order n given by `TriangleQuadratureFormula`. Make your code efficient by avoiding unnecessary function evaluations and use Matlab's vector notation where possible.

d) Create another function

```
v = GlobalQuadrature( f, p, e, t)
```

where f is a function handle and p, e, t describe the triangulation. The result v should be the approximate integral of f over the domain covered by the triangulation using `LocalTriangleQuadrature` on each triangle.

e) Download the example mesh file `mesh.mat` and load it into Matlab by using the command `load('mesh.mat').` and compute the integrals $\int_\Omega x^i y^i$ for $n \in \{1, \ldots, 4\}$ and $i \in \{1, \ldots, 4\}$ using `GlobalQuadrature`.

Have fun!