Fachbereich Mathematik & Informatik
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Exercise 7 for the lecture

Numerics III

SoSe 2018

http://numerik.mi.fu-berlin.de/wiki/SS_2018/NumericsIII.php

Due: Wed, 06-20-2018

Problem 1 (4 TP)
Let $\Omega \subset \mathbb{R}^n$ be a domain with a sufficiently smooth boundary. Consider the boundary value problem

$$\begin{aligned} -\alpha\Delta u + \beta \cdot \nabla u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

with $\alpha > 0$, $\beta \in \mathbb{R}^n$ and a given function $f \in C(\overline{\Omega})$.

a) Derive a weak formulation of this problem and show, that the resulting variational problem is well posed in the Sobolev space $H_0^1(\Omega)$.

b) How does the variational problem change, if β is a function in $C^1(\overline{\Omega})^n$, and under which condition do we have still a well posed problem?

Problem 2 (4 TP)
Consider the space $H^{\frac{1}{2}}(\partial\Omega) := \operatorname{tr}(H^1(\Omega))$.

a) Define a norm on $H^{\frac{1}{2}}(\partial\Omega)$, such that $\operatorname{tr} : H^1(\Omega)/\ker(\operatorname{tr}) \to H^{\frac{1}{2}}(\partial\Omega)$ is an isometry with respect to the quotient norm.

b) Show that $c\|v\|_{L^2(\partial\Omega)} \leq \|v\|_{H^{\frac{1}{2}}(\partial\Omega)} \; \forall v \in H^{\frac{1}{2}}(\partial\Omega)$.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.