Fachbereich Mathematik & Informatik
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Exercise 8 for the lecture

Numerics III

SoSe 2018

http://numerik.mi.fu-berlin.de/wiki/SS_2018/NumericsIII.php

Due: Wed, 06-27-2018

Problem 1 (4 TP)
In the lecture, an approximation of the variational problem

$$u \in H_0^1(\Omega): \quad a(u,v) = l(v) \quad \forall v \in H_0^1(\Omega)$$

was derived by using the finite element space S_h. The resulting variational problem

$$u_h \in S_h: \quad a(u_h,v) = l(v) \quad \forall v \in S_h$$

is rewritten as the linear system of equations

$$AU = b\ .$$

Show that A is positive definite if $a(\cdot,\cdot)$ is elliptic and that symmetry of $a(\cdot,\cdot)$ implies symmetry of A.

Problem 2 (4 TP)
Let us consider the boundary value problem

$$\begin{aligned} -(p(x)u')' + q(x)u &= f(x), \quad x \in (0,1) \qquad (P)\\ u(0) = u(1) &= 0, \end{aligned}$$

where $p, a \in C([0,1])$, $f \in L^2(0,1)$, $p(x) \geq p_0 > 0$, $q(x) \geq 0$. Consider the division of $\overline{\Omega = [0,1]}$ into $N \geq 2$ subintervals $[x_i \ldots, , x_{i+1}], i = 0, ..., N-1$, by the points $x_i = ih, i = 0, ..., N$, where $h = 1/N$,. What are the nodal first order finite element basis functions ϕ_i? What is the support of ϕ_i?
Furthermore, we define $V_h := span\{\phi_1, \ldots, \phi_{N-1}\}$, which is an $(N-1)-$dimensional subspace of $H_0^1(0,1)$. What is the finite element approximation of (P)?
Write the previous variational equation as a finite linear system with a symmetric positive definite matrix A i.e. in the form $AU = b$.

Calculate the entries of the matrix A explicitly.

Problem 3 (4 TP)
Let $T \subset \mathbb{R}^2$ be a triangle with the vertices $P_i = (x_i, y_i), i = 1, 2, 3$. Compute the affine transformation of the reference triangle T_0 to T, that maps $(0, 0)$ to (x_1, y_1), $(1, 0)$ to (x_2, y_2) and $(0, 1)$ to (x_3, y_3). Which geometric property of T is connected to the determinant of the matrix of the transform?
Consider a linear function u_h on the triangle T which at nodes has the given values $u_h(P_i)$. Find the formula for ∇u_h.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.