Fachbereich Mathematik & Informatik
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Exercise 9 for the lecture

Numerics III

SoSe 2018

`http://numerik.mi.fu-berlin.de/wiki/SS_2018/NumericsIII.php`

Due: Tue, 07-03-2018

Problem 1 (8 PP)

a) Make yourself familiar with the `Matlab` programmes `basis.m`, `quadrature.m` and `uniform_grid.m` on the homepage.

b) Write a `Matlab` programme `A = assemble_P1(grid, local_assem, Q)`, which assembles the global matrix $A_{i,j} = a(\lambda_j, \lambda_i)$ for the linear finite elements nodal basis $\{\lambda_i\}$ on the grid `grid`. The matrix should be calculated as a sum of element matrices assembled by the function `M = local_assem(T, B, Q)`. Thereby the columns of `T` give a triangle of the grid, `B = basis(1)` the local basis and `Q` a quadrature rule defined on the unit simplex. Test your programme by assembling the stiffness and the mass matrix for a uniform grid, using the local assemblers `assemble_stiff` and `assemble_mass` and appropriate quadrature rules.

c) Write a `Matlab` programme `[A,b] = assemble_dirichlet(grid, A, b, g)`, which „includes“Dirichlet boundary conditions, given by the function `function y = g(x)`, in the matrix `A` and the right-hand side `b`.

d) Use your programmes to approximate a solution of the problem

$$-\Delta u = f \quad \text{in } \Omega, \qquad u = g \quad \text{on } \partial\Omega$$

for

$$f(x) = \begin{cases} 0.2 & \text{for } |x - (0.5, 0.5)| \leq 0.2 \\ 0 & \text{else} \end{cases}$$

and $g = 0$ with linear finite elements on the unit square $\Omega = [0,1]^2$ and on the unit circle $\Omega = K_1(0)$, and visualize the solution with the `Matlab` command `trisurf`.

Advices:

- You can load a grid on the unit circle with the command `grid = load('circle')`, using the file 'circle' on the homepage.
- The right-hand side b can be assembled by linear interpolation of f, i.e. by evaluation at the grid points and multiplication with the mass matrix.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `adjurdjevac@mi.fu-berlin.de` with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.