Fachbereich Mathematik & Informatik
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Exercise 2 for the lecture

Numerics III

WS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, May 8th via Email

1. Exercise (3 TP)
Explain the physical meaning of Dirichlet, Neumann, and Cauchy boundary conditions for the heat equation resulting from energy conservation and for the nonlinear continuity equation equation resulting from car conservation.

2. Exercise (4 TP)
Consider the Diffusion equation

$$\Delta u = f \qquad \text{in } \Omega$$

with $f \in C(\overline{\Omega})$ and homogeneous Neumann boundary conditions.

a) Show that a solution $u \in C^2(\overline{\Omega})$ is not unique.

b) Show that there exists no solution $u \in C^2(\overline{\Omega})$, if $\int_\Omega f \, dx \neq 0$.

3. Exercise (4 TP)
Consider a domain $\Omega \subset \mathbb{R}^d$, $d = 1, 2$ with a disjoint decomposition $\partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_R$ of the boundary for Dirichlet, Neumann and Robin conditions for the differential equation

$$\begin{aligned} -\alpha \Delta u(x) &= f(x) && \forall x \in \Omega \\ u(x) &= 0 && \forall x \in \Gamma_D \\ \frac{\partial u}{\partial n}(x) &= g_N(x) && \forall x \in \Gamma_N \\ u(x) + \beta \frac{\partial u}{\partial n}(x) &= g_R(x) && \forall x \in \Gamma_R \end{aligned}$$

with $f \in C(\overline{\Omega})$ and $\beta > 0$ where n is the unit outer normal vector on $\partial\Omega$. Derive a variational equality

$$u \in V \qquad a(u, v) = l(v) \qquad \forall v \in V$$

for this PDE and specify the space V, the bilinear form a and the functional l.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.