Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Xingjian Zhang-Schneider

Exercise 3 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, May 15th via Email

1. Exercise (4 TP)

a) Let $\mathbb{R}^2_+ = \{(x,t) \mid x, t \in \mathbb{R}, t > 0\}$. Show that the Cauchy problem to find

$$u \in \{v \in C(\overline{\mathbb{R}^2_+}) \mid v_{xx} \in C(\mathbb{R}^2_+), v_t \in C(\mathbb{R}^2_+)\}$$

such that

$$u_{tt} = c u_{xx} \quad x \in \mathbb{R},\ t > 0, \qquad u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}$$

holds with given $u_0,\ u_1 \in C(\mathbb{R})$ and $c > 0$ has the d'Alembert solution

$$u(x,t) = \tfrac{1}{2}(u_0(x+ct) + u_0(x-ct)) + \tfrac{1}{2c}\int_{x-ct}^{x+ct} u_1(s)\, ds.$$

b) Show that the d'Alembert solution is unique.

2. Exercise (4 TP)

Let $\Omega = (0,\pi) \times (0,\pi)$. Show that the boundary value problem to find

$$u \in \{v \in C(\overline{\Omega}) \mid v_{xx} \in C(\Omega),\ v_y \in C(\Omega)\}$$

such that

$$u_y = u_{xx}, \quad (x,y) \in \Omega,$$

$$u(x,0) = \sin(x), \quad x \in (0,\pi),$$

$$u(x,y) = 0, \quad (x,y) \in \partial\Omega \setminus \{(x,0) \mid x \in (0,\pi)\}$$

is ill-posed.

3. Exercise (4 TP)
Classify the following PDEs.

a) $-\mathrm{div}(\alpha(x)\nabla u) = 0, \qquad \alpha \in C^1(\Omega), \quad \alpha(x) \geq \alpha_0 > 0.$

b) $\varepsilon \Delta u - \vec{\beta} \nabla u = 0, \qquad$ (i) for $\varepsilon \neq 0$, (ii) for $\varepsilon = 0$.

c) $u_{tt} - \Delta u = 0.$

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.