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Exercise 4 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, May 22th via Email

1. Exercise (4 TP)

a) Transform the Laplace equation $\Delta u(x,y) = 0$ for x, $y > 0$ in to polar coodinates.

b) Confirm or falsify that $u(r,\varphi) = r^{2/3}\sin(\frac{2}{3})$ a solution of the Laplace equation.

2. Exercise (4 TP)

Consider a domain $\Omega \subset \mathbb{R}^d$ and $G : (\overline{\Omega} \times \Omega) \setminus \{(x,a)|x = a\} \to \mathbb{R}$ such that each $G(\cdot, a)$ is a Green's function of first kind. Show that G is strictly positiv, i.e.,

$$G(x,a) > 0 \qquad \forall x, a \in \Omega, x \neq a$$

by using a Maximum principle.

3. Exercise (4 TP)

a) Let Ω be a bounded and open set in $\mathbb{R}^n$. If $u \in C^2(\Omega) \cup C(\bar{\Omega})$ is a harmonic function, then

$$\max_{\bar{\Omega}} u = \max_{\partial\Omega} u \quad \text{and} \quad \min_{\bar{\Omega}} u = \min_{\partial\Omega} u. \tag{1}$$

b) Let $u(x) = \log|x|$. Show that u is harmonic on $\mathbb{R}^2 \backslash \{0\}$.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjian@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.