Fachbereich Mathematik & Informatik
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Exercise 5 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, May 29th via Email (TP) and Friday June 05 th via Email (PP)

1. Exercise (4 TP)
Prove or disprove the following statement. The coefficient matrix A obtained from the Shortley-Weller method is symmetric, if and only if $\partial\Omega_h \subset \{(ih, jh) \mid i, j \in \mathbb{Z}\}$.

2. Exercise (4 TP)
Consider the grid Laplacian $\Delta_h U(x) = \sum_{y \in \overline{\Omega}_h} \alpha(x, y) U(y)$ for $x \in \Omega_h$ as obtained from the Shortley-Weller scheme. Then the weights $\alpha(x, y)$ have the properties

a) $a(x, x) < 0$, $\alpha(x, y) > 0$, $y \in Nb(x)$, $y \neq x$,

b) $\sum_{y \in \overline{\Omega}_h} \alpha(x, y) = 0$

for all $x \in \Omega_h$.

3. Exercise (8 PP)
Let $\Omega := [0, 1]^2$ and $f, g \in C(\overline{\Omega})$.

a) For each $N \in \mathbb{N}$ let $h := \frac{1}{N}$ and $\Omega_N := \big\{(ih, jh) \mid (i, j) \in \{0, \ldots, N\}^2\big\}$. Implement the Shortley-Weller method for the problem

$$\begin{aligned} -\Delta_h U(x) &= f(x) \text{ for each } x \in \Omega_N \\ U(x) &= g(x) \text{ for each } x \in \partial\Omega_N \end{aligned} \tag{1}$$

by writing functions

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A = OperatorAssembler( N )
F = FunctionalAssembler( f, g, N )
```

which assemble the coefficient matrix A and the right-hand side vector F of this discrete equation for given function handles `f` and `g`. The returned matrix should be stored in a sparse format.

b) Let

$$\begin{array}{lll} f_1 = 2(-x_1^2 - x_2^2 + x_1 + x_2) & g_1 = 0 & u_1 = (x_1^2 - x_1)(x_2^2 - x_2) \\ f_2 = -4 & g_2 = \|x\|^2 & u_2 = \|x\|^2 \\ f_3 = 2\pi^2 \sin(\pi x_1)\sin(\pi x_2) & g_3 = 0 & u_3 = \sin(\pi x_1)\sin(\pi x_2). \end{array}$$

Solve the discrete problem (1) given (f_k, g_k) and $N = 2^l$ for all $k \in \{1, 2, 3\}$ and $l \in \{2, \ldots, 8\}$. Plot the graphs of the discrete solutions U. In addition, plot the errors $\max_{x \in \Omega_N} |u_k(x) - U_k(x)|$ versus $h = \frac{1}{N}$ in a logarithmic scale with 1:1 aspect-ratio. You can use the command `axis equal` and the function `loglog`.

Remark It is necessary that you add comments to your code which explain your implementation.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjianz@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.