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Exercise 6 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, June 05th via Email

1. Exercise (4 TP)
Let $[a, b] \subseteq \mathbb{R}$ be an interval. Suppose $f \in C(\overline{I})$ and that $u \in C^2(\overline{I})$ is a solution of

$$-\Delta u = f, \qquad u|_{\{a,b\}} = 0.$$

Furthermore, let $N \in \mathbb{N}$ and $h := \frac{b-a}{N}$. For each $i \in \{0, \dots, N\}$ define $x_i := a + ih$ and for every $i \in \{1, \dots, N-1\}$ let

$$\lambda_i \colon \overline{I} \longrightarrow \mathbb{R}, \qquad x \longmapsto \begin{cases} \frac{1}{h}(x - x_{i-1}) & \text{for } x \in [x_{i-1}, x_i] \\ -\frac{1}{h}(x - x_{i+1}) & \text{for } x \in [x_i, x_{i+1}] \\ 0 & \text{for } x \notin [x_{i-1}, x_{i+1}]. \end{cases}$$

Prove for all $i \in \{1, \dots, N-1\}$ that

$$-\Delta_h u(x_i) := -\frac{u(x_{i-1}) - 2u(x_i) + u(x_{i+1})}{h^2} = \frac{\int_I f(x)\lambda_i(x)\,\mathrm{d}x}{\int \lambda_i(x)\,\mathrm{d}x} =: f_h(x_i).$$

Hint : Multiply the PDE by λ_i, then integrate and use partial integration.

2. Exercise (4 TP)
Consider the space

$$\mathcal{L}(V, W) = \{L : V \to W | L \text{ linear and bounded}\}.$$

Show that $\mathcal{L}(V, W)$ is a Banach space, if W is a Banach space.

3. Exercise (4 TP)
Let us consider the inner product space

$$H_C = \{v \in C^1(\Omega) \cap C(\bar{\Omega}) \,|\, v|_{\partial\Omega} = 0, \|v\|_{H^1(\Omega)} < \infty\},$$

equipped with the inner product $(\cdot, \cdot)_{H^1(\Omega)}$. Find a bilinear form on H_C, which is positive definite, but not H_C-elliptic.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjianz@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.