Fachbereich Mathematik & Informatik
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Exercise 7 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, June 12th via Email

1. Exercise (4 TP)

Let H be a Hilbert space with scalar product $(\cdot,\cdot)$ and $S \subset H$ a closed subspace. Show that

$$Pu = \operatorname*{argmin}_{v \in S} \left(\frac{1}{2}(v,v) - (u,v) \right)$$

defines an orthogonal projection $P : H \to S$, the so-called *Ritz projection*.

2. Exercise (4 TP)

Two norms $\|\cdot\|_A$ and $\|\cdot\|_B$ on a normed vector space V are equivalent if there are constants $c_1, c_2 > 0$ with

$$c_1\|v\|_A \le \|v\|_B \le c_2\|v\|_A \qquad \forall v \in V.$$

Show that $\|v\|_{L^2(\Omega)} + \|\nabla v\|_{L^2(\Omega)}$ is equivalent to the norm $\|v\|_1$ on $H^1(\Omega)$ given by

$$\|v\|_1 = \sqrt{(v,v)_1}, \qquad (u,v)_1 = (u,v)_{L^2(\Omega)} + (\nabla u, \nabla v)_{L^2(\Omega)}$$

with constants independent on the dimension and Ω.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjianz@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.