

Exercise 8 for the lecture

NUMERICS III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, June 19th via Email

1. Exercise (4 TP)

- a) Show that the following functions are contained in $\mathcal{D}'([-1, 1])$

$$f(x) = \sin(x), \quad g(x) = |x|, \quad h(x) = |x|^{1/2}.$$

- b) Compute the distributional derivatives of f , g , h . Which of the distributional derivatives are weak derivatives of f , g , h ?

2. Exercise (4 TP)

Is a classical solution of

$$\begin{aligned} -\Delta u &= f \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

always a weak solution of the corresponding variational formulation? Please provide a proof or a counterexample.

3. Exercise (4 TP)

- a) Derive a weak formulation of the boundary value problem

$$-\Delta u + \beta \cdot \nabla u = f \quad \text{in } \Omega$$

with $\beta \in \mathbb{R}^2$ and homogeneous Dirichlet boundary conditions.

- b) Show existence, uniqueness of a solution u for $f \in L^2(\Omega)$ and its continuous dependence on perturbations of f in $L^2(\Omega)$.

GENERAL REMARKS

You have to do the exercises in groups of up to 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjianz@gmail.com, with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advice for programming exercises on the homepage.