Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Xingjian Zhang-Schneider

Exercise 10 for the lecture

Numerics III

SS 2020

http://numerik.mi.fu-berlin.de/wiki/SS_2020/NumericsIII.php

Due: Friday, July 3rd for TP and July 10th for PP via Email

1. Exercise (4 TP)
We consider the variational problem

$$u \in H \qquad a(u,v) = l(v) \qquad \forall v \in H \tag{1}$$

with a Hilbert space H, a symmetric, H-elliptic bilinear form $a(\cdot,\cdot)$, and $l \in H'$. Let $\|\cdot\| = a(\cdot,\cdot)^{1/2}$ denote the energy norm, let $S \subset Q \subset H$ be closed subspaces of H, and let $u_S \in S$ and $u_Q \in Q$ be the Ritz approximations of u with respect to S and Q. Show that the saturation assumption

$$\|u - u_Q\| \leq \beta \|u - u_S\| \quad \text{with } \beta < 1 \tag{2}$$

is equivalent to the a posteriori error estimate

$$\|u - u_S\| \leq C \|u_Q - u_S\| \quad \text{with } C = \frac{1}{\sqrt{1-\beta^2}} \tag{3}$$

2. Exercise (8 PP, July 10th)

a) Make yourself familiar with the MATLAB programmes `basis.m`, `quadrature.m` and `uniform_grid.m` on the homepage.

b) Write a MATLAB programme `A = assemble_P1(grid, local_assem, Q)`, which assembles the global matrix $A_{i,j} = a(\lambda_j, \lambda_i)$ for the linear finite elements nodal basis $\{\lambda_i\}$ on the grid `grid`. The matrix should be calculated as a sum of element matrices assembled by the function `M = local_assem(T, B, Q)`. Thereby the columns of `T` give a triangle of the grid, `B = basis(1)` the local basis and `Q` a quadrature rule defined on the unit simplex. Test your programme by assembling the stiffness and the mass matrix for a uniform grid, using the local assemblers `assemble_stiff` and `assemble_mass` and appropriate quadrature rules.

c) Write a MATLAB programme `[A,b] = assemble_dirichlet(grid, A, b, g)`, which „includes“Dirichlet boundary conditions, given by the function `function y = g(x)`, in the matrix `A` and the right-hand side `b`.

d) Use your programmes to approximate a solution of the problem

$$-\Delta u = f \quad \text{in } \Omega, \qquad u = g \quad \text{on } \partial\Omega$$

for

$$f(x) = \begin{cases} 0.2 & \text{for } |x - (0.5, 0.5)| \leq 0.2 \\ 0 & \text{else} \end{cases}$$

and $g = 0$ with linear finite elements on the unit square $\Omega = [0,1]^2$ and on the unit circle $\Omega = K_1(0)$, and visualize the solution with the MATLAB command `trisurf`.

Advices:

- You can load a grid on the unit circle with the command `grid = load('circle')`, using the file 'circle' on the homepage.
- The right-hand side b can be assembled by linear interpolation of f, i.e. by evaluation at the grid points and multiplication with the mass matrix.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to xingjianz@gmail.com. with a subject starting by [NumericsIII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.