Inverse Problems | Exercise 10
Sommersemester 2022
Return your written solutions either in person or by email
to vesa.kaarnioja@fu-berlin.de by Monday 11 July, 2022, 12:15

1. Suppose $\rho_1 \sim \mathcal{N}(m_1, \sigma_1^2)$ and $\rho_2 \sim \mathcal{N}(m_2, \sigma_2^2)$. Solve the Kullback–Leibler divergence
$$d_{\mathrm{KL}}(\rho_1 \| \rho_2) = \int_{\mathbb{R}} \log\left(\frac{\rho_1(x)}{\rho_2(x)}\right) \rho_1(x)\,\mathrm{d}x.$$

2. Let $\rho \sim \mathcal{N}(0,1)$ and assume that the posterior density is given by
$$\rho^y(x) = 0.6 \cdot \rho(x-3) + 0.4 \cdot \rho(x+2).$$
Using MATLAB try to find the KL-optimal approximation for ρ^y in the class of $\mathcal{B} = \{\mathcal{N}(m,1) \mid m \in \mathbb{R}\}$, that is, find m that solves
   (a) $\inf_{m\in\mathbb{R}} d_{\mathrm{KL}}(\mathcal{N}(m,1)\|\rho^y)$
   (b) $\inf_{m\in\mathbb{R}} d_{\mathrm{KL}}(\rho^y\|\mathcal{N}(m,1))$

   What difference do you find between the two approximations?

   Hint: You are not required to do anything too sophisticated in this task. Any reasonable minimization procedure or quadrature rule is OK.

3. Show rigorously that $d_{\mathrm{KL}}(\rho_1\|\rho_2) = 0$ if and only if $\rho_1 = \rho_2$ a.e.