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Exercise 1 for the lecture

Numerics II

WS 2011/12

Due: till Thursday, 03. November 12 o'clock

Problem 1 (4 TP)

a) Rewrite the system

$$\begin{aligned} x_1' &= \alpha x_1 - \beta x_2 \\ x_2' &= \beta x_1 + \alpha x_2 \end{aligned}$$

as a scalar complex ode.

b) Derive conditions on α, β for stability and asymptotic stability of the fixed point $x^* = 0$.

c) Discuss the phase diagrams of solutions in the stable, asymptoticly stable and unstable case.

Problem 2 (3 TP + 1 PP)

a) Check numerically whether $x^* = 0$ is a stable fixed point of

$$\begin{aligned} x_1' &= x_1^3 \quad -x_2 \\ x_2' &= x_1 \end{aligned} \qquad (1)$$

or not. Use the `MATLAB`-functions `ode23` for approximation and `odephase2` to visualize the phase diagram.

b) Show that the fixed point $x^* = 0$ of (1) is not stable. Hint: Show blow-up of $V(x(t))$, $V(x) = x_1^2 + x_2^2$ in finite time.

Problem 3 (2 TP)
Consider the following system of odes

$$x'(t) = f(x(t)), \qquad f(x) = \begin{pmatrix} 1 - \frac{1}{2}(x_1 - x_2)^2 \\ \frac{1}{\sqrt{2}}(x_1 + x_2) \end{pmatrix}. \tag{2}$$

a) Calculate all fixed points of (2).

b) Discuss the (asymptotic) stability of these fixed points.

Problem 4 (4 TP)
Consider the following initial value problem

$$x'(t) = A(t)x(t), \qquad x(0) = (-\varepsilon, 0)^T, \tag{3}$$

with the matrix

$$A(t) = \begin{pmatrix} -1 + \frac{3}{2}\cos(t)^2 & 1 - \frac{3}{2}\cos(t)\sin(t) \\ -1 - \frac{3}{2}\cos(t)\sin(t) & -1 + \frac{3}{2}\sin(t)^2 \end{pmatrix}.$$

a) Calculate the eigenvalues of $A(t)$.

b) How does the spectral abscissa behave for varying t? What do you expect concerning the (asymptotic) stability of solutions?

c) Determine the solution $x(t)$ of (3) explicitly. How does $x(t)$ behave for $t \to \infty$? (Hint: x is a product of trigonometric functions and the exponential function.)