Department of Mathematics & Computer Science
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Maren-Wanda Wolf

Exercise 11 for the lecture

Numerics II

WS 2011/12

Due: till Thursday, 26. January 12 o'clock

Problem 1 (4 TP)
Show that a convergent linear iteration converges linearly if the system matrix and the preconditioner of the iteration are s.p.d..

Problem 2 (4 TP)
Let $A \in \mathrm{R}^{n,n}$ be symmetric positive definite and $b \in \mathrm{R}^n$.

a) Show that there is a strictly convex functional $J : \mathrm{R}^n \to \mathrm{R}$ such that the solution of the linear system $Ax = b$ is the minimizer of J.

b) Show that the Richardson iteration

$$x_{k+1} = x_k + \omega(b - Ax_k)$$

with $\omega \in \mathrm{R}$ is equivalent to the explicit Euler method for the gradient flow associated with J.

c) Show that there is an $\omega > 0$ such that the Richardson iteration converges to the solution of the linear system without using the results of Chapter 4 on numerical linear algebra.

d) Why is using an implicit Runge-Kutta method for the gradient flow not a good idea to solve the linear system ?

Problem 3 (4 TP)

a) Show that the parallel directional correction method associated with the Euclidean unit vectors e_i is the Jacobi method.

b) Show that the successive directional correction method associated with the Euclidean unit vectors e_i is the Gauß-Seidel method.

Problem 4 (4 TP)
Assume that $A \in \mathrm{R}^{n,n}$ is s.p.d.. Prove the convergence of the gradient method

$$x^{k+1} = x^k + \omega^k r^k, \qquad r^k = b - Ax_k, \qquad \omega^k = \frac{\langle r^k, r^k \rangle}{\langle Ar^k, r^k \rangle}.$$