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Exercise 2 for the lecture

Numerics II

WS 2011/12

Due: till Thursday, 10. November 12 o'clock

Problem 1 (2 TP)
Discuss the stability of the fixed point $x^* = 0$ of the following odes

a)

$$\begin{array}{rcrcl} x_1' &=& -x_1 &+& x_2^2 \\ x_2' &=& -\exp(x_1)x_2 && \end{array}$$

b)

$$\begin{array}{rcrcl} x_1' &=& \cos(x_1) &-& \exp(-x_2) \\ x_2' &=& x_1 x_2 && \end{array}$$

Problem 2 (2 TP + 2 PP)
We consider the initial value problem

$$x'(t) = \lambda |x(t)| x(t), \qquad t > 0, \qquad x(0) = x_0. \tag{1}$$

with the solution

$$x(t) = \frac{x_0}{1 - \lambda |x_0| t}$$

and the fixed point $x^* = 0$.

a) Is x^* stable or even asymptotically stable for $\lambda < 0$?

b) Solve the problem (1) numerically for $\lambda = -1000$ using the explicit `MATLAB`-solver `ode45`.

c) Discuss whether (1) is a stiff or a non-stiff problem.

Problem 3 (2 TP)

a) Prove that $x_{k+1} = Bx_k$ is stable, iff

$$\sup_{k \in \mathbb{N}} \|B^k\| < \infty.$$

b) Prove that $x_{k+1} = Bx_k$ is asymptotically stable, iff

$$\lim_{k \to \infty} \|B^k\| = 0.$$

Problem 4 (3 TP)

a) Compute the stability function for the implicit midpoint rule.

b) Show that the stability function for the Runge-Kutta-4 method is given by

$$R(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}.$$

c) Show that the stability function for the implicit trapezoidal rule is given by

$$R(z) = \frac{1 + \frac{z}{2}}{1 - \frac{z}{2}}.$$