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Exercise 7 for the lecture
Numerics II
WS 2011/12

Due: till Thursday, 15. December 12 o'clock

Problem 1 (5 TP)
A ball is kicked with initial horizontal speed $x_1'(0) = 80km/h$ from the top of the empire state building.

a) Compute and visualize the path of the ball for $t \in [0, T]$ with $T = 100s$.

b) How long does it take until the ball hits the street? Would it take longer, if $x_1'(0) = 160km/h$?

c) Compute the length of the ball's path.

Problem 2 (3 TP)
Show that

$$N = \begin{pmatrix} 0 & & & 0 \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{pmatrix} \in \mathrm{R}^{d \times d}$$

is nilpotent, i.e., $N^{d-1} \neq 0$ and $N^d = 0$.

Problem 3 (6 TP)
Consider the differential algebraic system

$$\begin{pmatrix} 0 & 0 & 0 \\ c & 0 & -c \\ 0 & 0 & 0 \end{pmatrix} x' = \begin{pmatrix} 1 & 0 & -1 \\ c & -\frac{1}{R} & \frac{1}{R} \\ 0 & 0 & 1 \end{pmatrix} x + \begin{pmatrix} -U \\ 0 \\ 0 \end{pmatrix}, \qquad x(0) = x_0 \qquad (1)$$

with $c, R, U > 0$.

a) Rewrite this system in normal form, i.e., as

$$y'(t) = Jy(t) + f, \qquad\qquad y(0) = y_0,$$
$$Nz'(t) = z(t) + g, \qquad\qquad z(0) = z_0.$$

Give f, g, J, N, y_0, z_0, and the transformation $T : x \mapsto (y, z)$.

b) How must x_0 be chosen such that there is a unique solution of (1) ?

Problem 4 (3 TP)

Show that the differentiation index of a linear differential algebraic system is invariant under equivalence transformations.