

Exercise 1 for the lecture  
**NUMERICS IV**  
WS 2012/13

**Due: till Wednesday, October 31, 2012, 14 o'clock**

**Problem 1** (3 TP)

Let  $\Gamma = \left( \begin{array}{c} X(\theta) \\ Y(\theta) \end{array} \right)_{\theta \in [0,1]}$  with  $X, Y \in C^2[0, 1]$ .

Show that  $\Gamma$  is orientable or find a counter example.

**Problem 2** (3 TP)

Let  $\Gamma = \left( \begin{array}{c} X(\theta) \\ Y(\theta) \end{array} \right)_{\theta \in [0,1]}$  with  $X, Y \in C^2[0, 1]$  and  $X'(\theta)^2 + Y'(\theta)^2 > 0$  for all  $\theta \in [0, 1]$ .

Show that the curvature  $\kappa$  of  $\Gamma$  is given by

$$\kappa = \frac{X'Y'' - Y'X''}{((X')^2 + (Y')^2)^{\frac{3}{2}}}.$$

**Problem 3** (3 TP)

Show that the solution of the mean curvature flow of the unit circle is given by

$$X(\theta, t) = \sqrt{1 - 2t} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \quad 0 \leq t \leq \frac{1}{2}.$$

Hint: Use the ansatz  $X(\theta, t) = R(t)X(\theta, 0)$ .