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Exercise 10 for the lecture

# Numerics IV

WS 2012/2013

**Due: till Wednesday, January 16, 2013, 14 o'clock**

**Problem 1** (3 TP)
Show that $u(x) = |x| + 1$ is a viscosity solution of

$$-|u'(x)| + 1 = 0, \quad u(-1) = u(1) = 0$$

but not of

$$|u'(x)| - 1 = 0, \quad u(-1) = u(1) = 0$$

in $(-1, 1)$.

**Problem 2** (4 TP)
Show that $u(x) = -|x| + 1$ is the unique viscosity solution of

$$|u'(x)| = 1 \quad \in (-1, 1), \qquad u(-1) = u(1) = 0.$$

**Problem 3** (4 TP)
Show that, if $u$ is a viscosity solution of a second order pde

$$F(x, u, Du, D^2u) = 0, \quad x \in \Omega \subset \mathrm{R}^n,$$

then $-u$ solves in the viscosity sense $-F(-x, -u, -Du, -D^2u) = 0$.