Department of Mathematics & Computer Science
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Maren-Wanda Wolf

Exercise 11 for the lecture

Numerics IV

WS 2012/2013

Due: till Wednesday, January 30, 2013, 14 o'clock

Problem 1 (3 extra TP + 5 extra PP + 3 extra PP)
Consider the regularised mean curvature flow equation for level sets

$$\begin{aligned} u_t^\varepsilon - \Delta u^\varepsilon + \frac{\nabla u^\varepsilon}{Q_\varepsilon} \nabla^2 u^\varepsilon \frac{\nabla u^\varepsilon}{Q_\varepsilon} &= 0 && \text{in } \Omega \times (0,T) \\ u^\varepsilon &= 1 && \text{on } \partial\Omega \times (0,T) \\ u^\varepsilon(\cdot,0) &= u_0 && \text{in } \Omega, \end{aligned} \tag{1}$$

with $\varepsilon > 0$ and $Q_\varepsilon = \sqrt{\varepsilon^2 + |\nabla u^\varepsilon|^2}$.

a) Use the results from chapter 4 to derive a discretisation of problem (1).

b) Implement the discrete scheme from a) in MATLAB as a `function [u, t] = MCFlevelSet (N, tau, T, eps, u0)`, where `N`, `tau`, `T`, `eps`, and `u0` denote the number of nodes in one direction of the space grid, the time step size, the final time, the regularisation parameter and the initial value, respectively.

c) Use your program to approximate the solution of (1) with

$$u_0(x,y) = (x^2+y^2)^2 - 2(x^2-y^2) \quad \text{in } \Omega = [-1.5, 1.5]^2$$

$N = 100$, $\varepsilon = 0.1, 0.01$ and suitable τ.