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Exercise 12 for the lecture

# Numerics IV

WS 2012/2013

**Due: till Wednesday, February 6, 2013, 14 o'clock**

**Problem 1** (2 TP)
Show that the Cahn-Hilliard eqution is thermodynamically consistent, i.e.

$$\mathcal{E}(u(t)) \leq \mathcal{E}(u(t')) \quad \text{for } t' \geq t,$$

and mass conserving, i.e. $\int_\Omega u(x,t) = \text{const}$.

**Problem 2** (4 TP)
For fixed $x \in \Omega$, let $v$ be a solution of

$$\varepsilon v'(t) = -\frac{1}{\varepsilon}\Phi'(v(t)), \quad v(0) = v_0,$$

where $\Phi$ is a double well potential with minima in $-1$ and 1. Show that

$$v(t) \to \begin{cases} 1 & v_0 > 0 \\ -1 & v_0 < 0. \end{cases}$$

**Problem 3** (2 TP)
Let $H$ be a Hilbertspace and $\mathcal{K} \subset H$ closed. Show that the indicator functional $\chi_\mathcal{K}$,

$$\chi_\mathcal{K}(v) = \begin{cases} 0 & v \in \mathcal{K} \\ +\infty & v \notin \mathcal{K} \end{cases}$$

is lower semicontinuous.

**Problem 4** (3 TP)
Find a Hilbertspace $H$, an $H$-elliptic bilinearform $a(\cdot,\cdot)$ and a functional $\Phi : H \to \mathrm{R} \cup \{+\infty\}$, such that

a) $\Phi$ is convex and proper but not lower semicontinuous,

b) the minimisation problem

$$u \in H : \quad \frac{1}{2}a(u,u) + \Phi(u) \leq \frac{1}{2}a(u,v) + \Phi(v) \quad \forall v \in H$$

has no solution.