Department of Mathematics & Computer Science
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Maren-Wanda Wolf

Exercise 2 for the lecture

Numerics IV

SoSe 2012

Due: till Wednesday, November 7, 2012, 14 o'clock

Problem 1 (4 TP)
Let $\Gamma \subset \mathrm{R}^{n+1}$ and $W \subset \mathrm{R}^{n+1}$ an open neighbourhood of Γ.

a) For $f, g \in C^1(W, \mathrm{R})$ prove the product rule

$$\nabla_\Gamma(fg) = f\nabla_\Gamma g + g\nabla_\Gamma f$$

where $\nabla_\Gamma f = \nabla f - (\nabla f \cdot \nu)\nu = \big(\underline{D}_1 f, \ldots, \underline{D}_{n+1} f\big)^T$.

b) For $f \in C^1(W, \mathrm{R}^m)$ define

$$D_\Gamma f := \begin{pmatrix} \underline{D}_1 f_1 & \ldots & \underline{D}_{n+1} f_1 \\ \vdots & & \vdots \\ \underline{D}_1 f_m & \ldots & \underline{D}_{n+1} f_m \end{pmatrix}.$$

Show that $D_\Gamma f = Df(I - \nu\nu^T)$, where Df is the usual Jacabian of f.

c) For $f \in C^1(W, \mathrm{R}^m)$ and $g \in C^1(\mathrm{R}^m, \mathrm{R})$ derive a chain rule for D_Γ.

Problem 2 (2 TP)
Show that the curvature matrix $(H_{jk})_{j,k=1}^{n+1}$, definde by

$$H_{jk} = \underline{D}_j \nu_k, \quad j, k = 1 \ldots, n+1$$

is symmetric.

Problem 3 (3 TP)
Prove the following properties of the oriented distance function $d : \mathrm{R}^{n+1} \to \mathrm{R}$:

a) d is globally Lipschitz and $\exists \delta > 0: \; d \in C^2(\Gamma_\delta), \; \Gamma_\delta = \{x \in \mathrm{R}^{n+1} \mid |d(x)| < \delta\}$

b) $\forall x \in \Gamma_\delta \; \exists a(x) \in \Gamma: \; x = a(x) + d(x)\nu(a(x))$

c) $|\nabla d(x)| = 1$

Problem 4 (3 TP)
Let Γ be given as the zero-levelset of the oriented distance function $d : \mathrm{R}^{n+1} \to \mathrm{R}$, i.e.

$$\Gamma = \{x \in \mathrm{R}^{n+1} \mid d(x) = 0\}.$$

Prove that

$$H(x) = \Delta d(x), \quad x \in \Gamma.$$