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Exercise 4 for the lecture

Numerics IV

SoSe 2012

Due: till Wednesday, November 21, 2012, 14 o'clock

Problem 1 (4 TP)
Let $X : [0,T] \times \Gamma \to \mathrm{R}^{n+1}$ be a parametrisation of $\Gamma(t)$, $V(t,\theta) = \frac{\partial}{\partial t}X(t,\theta)$ its velocity vector field and $DX(t,\theta)$ its Jacobian. Denote $\gamma(t) = \det(DX(t,\theta))$ and $\omega(t) = \gamma(t)\|DX(t,\theta)^{-T} \cdot \nu\|$, where ν is the unit normal vector in $X(t,\theta)$.

a) Prove
$$\frac{\partial}{\partial t}\gamma(t) = \nabla \cdot V(t)\gamma(t).$$

b) Prove
$$\frac{\partial}{\partial t}\omega(0) = \nabla_\Gamma \cdot V(0).$$

Problem 2 (6 TP)

a) Let Ω be a domain in R^{n+1}, $X : [0,T] \times \Omega \to \mathrm{R}^{n+1}$ a parametrisation of $\Omega(t)$ and let J be a functional of Ω. Then the shape derivative of J at Ω in the direction of a vector field $V = V(0,\theta)$ is given by:
$$J'(\Omega)(V) = \lim_{t \downarrow 0} \frac{1}{t}\left(J(\Omega(t)) - J(\Omega)\right).$$
Compute $J'(\Omega)(V)$ for
$$J(\Omega) = \int_\Omega y\,dx$$
and $V(t,\theta) = \frac{\partial}{\partial t}X(t,\theta)$.

b) Let Γ be a C^1-hypersurface in R^{n+1}, $X : [0,T] \times \Gamma \to \mathrm{R}^{n+1}$ a parametrisation of $\Gamma(t)$ and let J be a functional of Γ. Then the shape derivative of J at Γ in the direction of a vector field $V = V(0,\theta)$ is given by:

$$J'(\Gamma)(V) = \lim_{t \downarrow 0} \frac{1}{t} \left(J(\Gamma(t)) - J(\Gamma) \right).$$

Compute $J'(\Gamma)(V)$ for

$$J(\Gamma) = \int_\Gamma y \, d\sigma$$

and $V(t,\theta) = \frac{\partial}{\partial t} X(t,\theta)$.