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Exercise 6 for the lecture

Numerics IV

SoSe 2012

Due: till Wednesday, December 12, 2012, 14 o'clock

Problem 1 (3 TP + 2 extra TP + 5 extra PP + 2 extra TP)
Consider the parametric equation

$$X_t = \frac{X_{\theta\theta}}{|X_\theta|^2}. \tag{1}$$

a) Show that (1) leads to the same evolution in normal direction as

$$X_t = \frac{1}{|X_\theta|}\left(\frac{X_\theta}{|X_\theta|}\right)_\theta,$$

i.e. (1) defines a curve evolving in normal direction with a normal velocity being given by the curvature.

b) (Zusatzaufgabe)
Derive a weak formulation of (1) and dicretise in space and time, using linear finite elements and an implicit euler scheme, respectively.

c) (Zusatzaufgabe)
Implement the discrete scheme derived in b) in Matlab as `function [X, t] = CurveShorteningWithTanMotion (N, tau, T, X0)`, where `N`, `tau`, `T`, and `X0` denote the number of nodes in the space grid, the time step size, the final time $T = 0.5$ and the initial value, the unit circle given as function from I to R^2, respectively. Test your programm with suitable parameters and compare your results with the solution of `function [X, t] = CurveShorteningImplicit (N, tau, T, X0)` from exercise 5.

d) (Zusatzaufgabe)
Compare the Ansatz considered here, with the ideas of Barrett, Garcke and Nürnberg published in *Numerical approximation of gradient flows for closed curves in* R^d.

Problem 2 (5 TP)
Let T_0 denote the reference triangle let and $\hat{I} : H^2(T_0) \to L^\infty(T_0)$ denote linear interpolation in the vertices. Then the interpolation error satisfies

$$\|f - \hat{I}f\|_{L^2(T_0)} \leq c_0 \|\nabla f\|_{L^2(T_0)}.$$

a) Derive a quadrature rule Q_0 on the reference triangle T_0 such that the quadrature error satisfies

$$|\int_{T_0} f dx - Q_0(f)| \leq \frac{1}{2} c_0 \|\nabla f\|_{L^2(T_0)}.$$

b) Derive a quadrature rule Q_T on a triangle $T \subset \mathrm{R}^2$ with $\mathrm{diam} T \leq h$ such that the quadrature error satisfies

$$|\int_T f dx - Q_T(f)| \leq Ch^2 \|\nabla f\|_{L^2(T)}.$$

with C depending only on c_0 and the interior angles of T.

c) Derive a quadrature rule Q_T^3 on a triangle $T \subset \mathrm{R}^3$ with $\mathrm{diam} T \leq h$ such that the quadrature error satisfies

$$|\int_T f dx - Q_T^3(f)| \leq Ch^2 \|\nabla f\|_{L^2(T)}.$$

with C depending only on c_0 and the interior angles of T.