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Exercise 8 for the lecture

Numerics IV

SoSe 2012

Due: till Wednesday, December 19, 2012, 14 o'clock

Problem 1 (3 TP)
Assume that $I_h : H^2(\Omega) \to S$ satifies $\|v - I_h v\|_0 \le ch^2 \|v\|_2$, $\|\nabla(v - I_h v)\|_0 \le ch\|v\|_0$ and let $\nu = \frac{1}{Q}\begin{pmatrix}\nabla u \\ -1\end{pmatrix}$, $Q = \sqrt{1 + |\nabla u|^2}$, $\nu_h = \frac{1}{Q_h}\begin{pmatrix}\nabla I_h u \\ -1\end{pmatrix}$, $Q_h = \sqrt{1 + |\nabla I_h u|^2}$ with $u \in H^2(\Omega)$. Show that

$$\int_\Omega |\nu - \nu_h|^2 Q_h \le ch^2.$$

Problem 2 (4 TP)
Let $u \in H^1\left((0,T), H^2(\Omega)\right)$ and ν, Q as in Problem 1. Show that

a) $Q_t \le |\nabla u_t|$

b) $\nu_t Q \le 2|\nabla u_t|$

Problem 3 (5 TP)
Let

$$\alpha(t) + \beta'(t) \le C_1 \beta(t) + C_2, \quad C_1, C_2 > 0.$$

Show a Gronwall lemma of the form

$$\int_0^t \alpha(s)\,\mathrm{d}s + \beta(t) \le \frac{1}{C_1} \exp(C_1 t)\left(C_2 + \beta(0)\right).$$