Department of Mathematics & Computer Science
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Maren-Wanda Wolf

Exercise 9 for the lecture

Numerics IV

SoSe 2012

Due: till Wednesday, January 16, 2013, 14 o'clock

Problem 1 (5 extra TP)

Prove that the fully discrete scheme for the mean curvature flow of graphs, given by

$$\frac{1}{\tau}\int_{\Omega_h} \frac{u_h^{m+1}\varphi_h}{Q_h^m} + \int_{\Omega_h} \frac{\nabla u_h^{m+1}\cdot\nabla\varphi_h}{Q_h^m} = \frac{1}{\tau}\int_{\Omega_h} \frac{u_h^m\varphi_h}{Q_h^m}, \tag{1}$$

with $Q_h^m = \sqrt{1+|\nabla u_h^m|^2}$, is stable in the sence that the solution u_h^m, $0 \le m \le M \le \frac{T}{\tau}$, of (1) satisfies, for every $m \in \{1,\dots,M\}$

$$\tau \sum_{k=0}^{m-1} \int_{\Omega_h} |V_h^k|^2 Q_h^k + \int_{\Omega_h} Q_h^m \le \int_{\Omega_h} Q_h^0$$

where $V_h^k = -\frac{(u_h^{k+1}-u_h^k)}{\tau Q_h^k}$ is the discrete normal velocity.

Problem 2 (3 extra TP + 5 extra PP)

Consider the minimal surface equation

$$\begin{aligned} -\nabla\cdot\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) &= f \qquad \text{in } \Omega \\ u &= g \qquad \text{on } \partial\Omega. \end{aligned} \tag{2}$$

a) Give the weak formulation of (2), discretise it using linear finite elements and derive an iterative method to solve the discrete nonlinear problem.
Hint: gradient flow.

b) Implement the iterative scheme from a) in Matlab and use your program to approximate the solution of (2) for $f = 0$ and $g = \sin(x)\sin(y)$ on the square $\Omega = [-1,1]^2$.