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Exercise 1 for the lecture

Fast solvers for nonsmooth PDEs

Winter term 2014

http://numerik.mi.fu-berlin.de/wiki/WS_2014/FastSolvers.php

Due: Thu, 2014-11-06, 10:15

Problem 1 (4 theory points)
For a bounded open domain Ω define

$$C^k(\Omega) := \{u : \Omega \to \mathbb{R} : u \text{ is } k \text{ times continuously differentiable}\},$$
$$C^k(\overline{\Omega}) := \{u \in C^k(\Omega) : D^\alpha u \text{ has a continuous extension to } \overline{\Omega} \text{ for } |\alpha| \leq k\}.$$

Show that $(C^1([0,1]), \|\cdot\|_1)$ with $\|u\|_1 = \|u\|_{L^2} + \|u'\|_{L^2}$ is an incomplete normed space.

Problem 2 (4 theory points)
Two norms $\|\cdot\|_A$ and $\|\cdot\|_B$ on a normed vector space V are equivalent if there are constants $c_1, c_2 > 0$ with

$$c_1\|v\|_A \leq \|v\|_B \leq c_2\|v\|_A \qquad \forall v \in V.$$

Show that $\|v\|_{L^2(\Omega)} + \|\nabla v\|_{L^2(\Omega)}$ is equivalent to the norm $\|v\|_1$ on $H^1(\Omega)$ given by

$$\|v\|_1 = \sqrt{(v,v)_1}, \qquad (u,v)_1 = (u,v)_{L^2(\Omega)} + (\nabla u, \nabla v)_{L^2(\Omega)}$$

with constants independent on the dimension and Ω.

Problem 3 (4 theory points)
Let Ω be a bounded open domain with Lipschitz-boundary. Show that there is a constant C only depending on Ω such that

$$\|u\|^2_{L^2(\Omega)} \leq C\Big(\|\nabla u\|^2_{L^2(\Omega)} + (u, \mathbf{1})^2_{L^2(\Omega)}\Big)$$

where $\mathbf{1}$ is the constant function being 1 everywhere.

Please turn around!

Problem 4 (4 theory points)

a) Show that the parallel directional correction method associated with the Euclidean unit vectors e_i is the Jacobi method.

b) Show that the successive directional correction method associated with the Euclidean unit vectors e_i is the Gauß-Seidel method.