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Exercise 7 for the lecture

Fast solvers for nonsmooth PDEs

Winter term 2014

`http://numerik.mi.fu-berlin.de/wiki/WS_2014/FastSolvers.php`

Due: Thu, 2014-12-18, 12:15

Problem 1 (8 programming points)
For $A \in \mathbb{R}^{n\times n}$ s.p.d. and $b, \underline{\psi} \in \mathbb{R}^n$ we consider the minimization problem

$$u = \arg\min_{v\in\mathbb{R}^n} \frac{1}{2}\langle Av, v\rangle - \langle b, v\rangle$$

and a given hierarchy of subspaces $V_1 \subset \cdots \subset V_m = \mathbb{R}^m$, $\dim V_k = n_k$.

a) Implement a linear multigrid step with Gauß–Seidel smoother as

```
v = mg_step(r,AA,PP)
```

computing the sum of all coarse grid corrections `v` for the residual `r`. Here `AA` is a cell arrays of size m containing the representations $A_k \in \mathbb{R}^{n_k\times n_k}$ of A for all levels $k = 1,\dots,m$ and `PP` is a cell arrays of size $m-1$ containing the prolongation operators $P_k \in \mathbb{R}^{n_{k+1}\times n_k}$ for all levels $k = 1,\dots,m-1$.

b) Implement a linear multigrid solver as

```
x = mg_solver(A,b,x0,PP,maxnu)
```

doing `maxnu` linear multigrid steps. Here `A` is the fine grid matrix $A \in \mathbb{R}^{n\times n}$

c) Test your method using the test suite provided on the lecture homepage.

d) Compare the convergence rate of the multigrid method and a pure Gauß–Seidel method for varying mesh size h.

Take care to have optimal complexity for sparse matrices in all implementations.

Problem 2 (8 programming points)
For $A \in \mathbb{R}^{n\times n}$ s.p.d., $b, \underline{\psi}, \overline{\psi} \in \mathbb{R}^n$ with $\underline{\psi} \le \overline{\psi}$, and $K = \{v \in \mathbb{R}^n \,|\, \underline{\psi} \le \overline{\psi}\}$ we consider the minimization problem

$$u = \arg\min_{v\in K} \frac{1}{2}\langle Av, v\rangle - \langle b, v\rangle$$

and a given hierarchy of subspaces $V_1 \subset \cdots \subset V_m = \mathbb{R}^m$, $\dim V_k = n_k$.

a) Implement a monotone multigrid step with projected Gauß–Seidel smoother as

```
v = mmg_step(r,lower,upper,AA,PP)
```

computing the sum of all coarse grid corrections `v` for the residual `r` and defect obstacles `lower` $\leq$ `upper`. Here `AA` is a cell arrays of size m containing the representations $A_k \in \mathbb{R}^{n_k \times n_k}$ of A for all levels $k = 1, \dots, m$ and `PP` is a cell arrays of size $m-1$ containing the prolongation operators $P_k \in \mathbb{R}^{n_{k+1} \times n_k}$ for all levels $k = 1, \dots, m-1$.

b) Implement a monotone multigrid solver as

```
x = mmg_solver(A,b,lower,upper,x0,PP,maxnu)
```

doing `maxnu` linear multigrid steps for obstacles `lower` $\leq$ `upper`. Here `A` is the fine grid matrix $A \in \mathbb{R}^{n \times n}$

c) Test your method using the test suite provided on the lecture homepage.

d) Compare the convergence rate of the multigrid method and a pure projected Gauß–Seidel method for varying mesh size h.

Take care to have optimal complexity for sparse matrices in all implementations.