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Exercise 8 for the lecture

Fast solvers for nonsmooth PDEs

Winter term 2014

http://numerik.mi.fu-berlin.de/wiki/WS_2014/FastSolvers.php

Due: Thu, 2015-01-08, 12:15

Problem 1 (10 programming + 4 theory points)
For $A \in \mathbb{R}^{n\times n}$ s.p.d., $b, \underline{\psi}, \overline{\psi} \in \mathbb{R}^n$ with $\underline{\psi} \leq \overline{\psi}$, and $K = \{v \in \mathbb{R}^n \,|\, \underline{\psi} \leq \overline{\psi}\}$ we consider the minimization problem

$$u = \arg\min_{v\in K} \frac{1}{2}\langle Av, v\rangle - \langle b, v\rangle$$

and a given hierarchy of subspaces $V_1 \subset \cdots \subset V_m = \mathbb{R}^m$, $\dim V_k = n_k$.

a) Extend the monotone multigrid step by an additional parameter to

```
v = tmmg_step(r,lower,upper,AA,PP,truncate)
```

where `truncate` is a flag that allows to switch on truncation of basis functions.

b) Extend the monotone multigrid solver by an additional parameter to

```
x = tmmg_solver(A,b,lower,upper,x0,PP,maxnu,truncateCycle)
```

switching on truncation of basis functions every `truncateCycle`-th iteration step for `truncateCycle`> 0, i.e.,

- `truncateCycle`$= 0$ resembles the classic monotone multigrid method (MMG),
- `truncateCycle`$= 1$ is the truncated monotone multigrid method (TMMG),
- `truncateCycle`$= 2$ is a hybrid version (HMMG) alternating between MMG and TMMG steps.

c) Test your method using the test suite provided on the lecture homepage.

d) Extend the test suite to allow initial iterates from nested iteration.

e) Compare iteration history and convergence rate of MMG, TMMG, and HMMG for trivial initial iterates and nested iteration.

f) Give a $1d$ example where you can prove that $\|u^\nu - u\|_A \geq 1 - C\frac{\nu}{n}$ holds for the TMMG method with bad initial iterates. Illustrate this example numerically and compare with MMG and HMMG and explain the results.

Take care to have optimal complexity for sparse matrices in all implementations.