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Exercise 1 for the lecture

# Numerics II

WS 2014/15

**Due: till Tuesday, 28. October**

**Problem 1**

Consider the differential equation

$$x'(t) = \lambda x(t) + \cos(t)e^{\lambda t}, \qquad t > 0 \tag{1}$$

with a real parameter $\lambda$ and initial value $x(0) = x_0 \in \mathrm{R}$.

a) Rewrite equation (1) to an autonomous equation. (Hint: choose $y = (x, t)$).

b) Investigate existence and uniqueness of solutions with respect to $\lambda$. Do *not* use part c).

c) Find a closed representation of the solution $x$. How does $x(t)$ behave for $t \to \infty$ in dependence of $\lambda$ and $x_0$?

d) Calculate the Wronski matrix and give the pointwise condition number of the differential equation (1).

**Problem 2** (6 PP)

a) Implement a `matlab` programm `function [x, t] = RungeKuttaEx(f, x0, I, tau, b, A)`, which performs an explicit Runge-Kutta method, given by `b` and `A` from the Butcher scheme and the timestep `tau`, for the autonomous initial value problem

$$x'(t) = f(x(t)), \qquad t \in (\mathtt{I(1)}, \mathtt{I(2)}] \qquad x(\mathtt{I(1)}) = x_0$$

with right hand side $f : \mathrm{R}^d \to \mathrm{R}^d$ and initial value $x_0 \in \mathrm{R}^d$.

The return value `x` should contain the solution and `t` the associated points in time as a vector.

b) Calculate the solution of the autonomous version of (1) from problem 1 on the intervall $[0, 10]$ with initial value $x_0 = 0$. Use Runge-Kutta-4, which is given by the Butcher scheme

$$\begin{array}{c|cccc} & 0 & & & \\ & 1/2 & 0 & & \\ & 0 & 1/2 & 0 & \\ & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 1/3 & 1/3 & 1/6. \end{array}$$

Plot your solution for $\lambda = -10, -100, -1000$ and timesteps $\tau = 0.001, 0.1$. Additionally, for each $\lambda$ plot the discretisation error over $\tau = 0.001 + k\,0.001$ for $k = 0, ..., 29$. Interprete your results.

**Problem 3**

Consider the following system of linear differential equations

$$x'(t) = Ax(t), \quad t > 0 \tag{2}$$

with the symmetric matrix

$$A = \begin{pmatrix} \frac{-1591}{25} & 0 & \frac{1212}{25} \\ 0 & -1 & 0 \\ \frac{1212}{25} & 0 & \frac{-884}{25} \end{pmatrix}.$$

a) Calculate all solutions of (2).

b) Give the solutions for the initial values $x_1(t_0) = (\frac{3}{5}, 0, \frac{4}{5})$ and $x_2(t_0) = (-\frac{4}{5}, 0, \frac{3}{5})$ and sketch their trajectories.