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Exercise 10 for the lecture

Numerics II

WS 2014/15

Due: till Tuesday, 20. January

Problem 1

a) Show that the symplectic Euler method is symplectic.

b) Show that the trapezoidal rule is symplectic if the Hamiltonian is quadratic, i.e. $H(y) = y^T C y$ holds with a symmetric real matrix C.

c) Show that the trapezoidal rule is not symplectic in general.

Problem 2

Consider the system $q' = p$; $p' = f(q)$.
The explicit one-step method given by

$$\begin{aligned} p_{n+\frac{1}{2}} &= p_n + \frac{\tau}{2} f(q_n) \\ q_{n+1} &= q_n + \tau p_{n+\frac{1}{2}} \\ p_{n+1} &= p_{n+\frac{1}{2}} + \frac{\tau}{2} f(q_{n+1}) \end{aligned}$$

is called Störmer-Verlet method.

Show that the Störmer-Verlet method is symplectic and has second order.

Problem 3

Consider the pendulum equation in polar coordinates

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = \begin{pmatrix} \frac{1}{m} p \\ -m \frac{g}{r_0} \cos q \end{pmatrix} \qquad \begin{pmatrix} q \\ p \end{pmatrix}(0) = \begin{pmatrix} q_0 \\ p_0 \end{pmatrix}$$

where q, g, m, and r_0 denote the angle, the gravity, the mass and the radius, respectively.

a) Implement the symplectic Euler method for this equation in `matlab` as `function [p, q, t] = SymplecticEuler(m, g, r0, p0, q0, I, tau)`, where `(m, g, r0)`, `(p0, q0)`, `I`, and `tau` denote the problem parameters, the initial values, the time interval and the step size, respectively.

b) Test your program with the radius $r_0 = 10cm$, the mass $m = 100g$, and the gravity of the moon. Use the time interval $[0s, 20s]$ with the initial value $p_0 = 0\frac{kgm}{s}$, $q_0 = 0m$ for various time step sizes.

c) Plot the solution in the phase space, the solution in Euclidean coordinates, and the associated Hamiltonian H.

d) Show that the radius $r = \sqrt{x_1^2 + x_2^2}$ is preserved if $g > 0$.