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Exercise 2 for the lecture

Numerics II

WS 2014/15

Due: till Tuesday, 4. November

Problem 1

Show that, the solution

$$x(t) = \frac{x_0 p \exp(pt)}{p + (\exp(pt) - 1)kx_0}, \quad p, k > 0$$

of the bacteria population problem

$$x' = px - kx^2, \quad t > 0, \quad x(0) = x_0$$

is not stable for $x_0 = 0$.

Problem 2

a) Rewrite the system

$$\begin{aligned} x_1' &= \alpha x_1 - \beta x_2 \\ x_2' &= \beta x_1 + \alpha x_2 \end{aligned}$$

as a scalar complex ode.

b) Derive conditions on α, β for stability and asymptotic stability of the fixed point $x^* = 0$.

c) Discuss the phase diagrams of solutions in the stable, asymptoticly stable and unstable case.

Problem 3

Consider the following system of odes

$$x'(t) = f(x(t)), \qquad f(x) = \begin{pmatrix} 1 - \frac{1}{2}(x_1 - x_2)^2 \\ \frac{1}{\sqrt{2}}(x_1 + x_2) \end{pmatrix}. \tag{1}$$

a) Calculate all fixed points of (1).

b) Discuss the (asymptotic) stability of these fixed points.

Problem 4

Consider the following initial value problem

$$x'(t) = A(t)x(t), \qquad x(0) = (-\varepsilon, 0)^T, \tag{2}$$

with the matrix

$$A(t) = \begin{pmatrix} -1 + \frac{3}{2}\cos(t)^2 & 1 - \frac{3}{2}\cos(t)\sin(t) \\ -1 - \frac{3}{2}\cos(t)\sin(t) & -1 + \frac{3}{2}\sin(t)^2 \end{pmatrix}.$$

a) Calculate the eigenvalues of $A(t)$.

b) How does the spectral abscissa behave for varying t? What do you expect concerning the (asymptotic) stability of solutions?

c) Determine the solution $x(t)$ of (2) explicitly. How does $x(t)$ behave for $t \to \infty$? (Hint: x is a product of trigonometric functions and the exponential function.)