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Exercise 3 for the lecture

Numerics II

WS 2014/15

Due: till Tuesday, 11. November

Problem 1
Consider the one-dimensional odes

a) $x' = x - x^2$

b) $x' = -x + 4x^3 - x^5$

c) $x' = \begin{cases} 0, & x = 0 \\ -x^3 sin(\frac{1}{x}), & x \neq 0. \end{cases}$

Find all fixed points and decide wether they are stable or unstable?

Problem 2
Consider the following system of odes

$$\begin{aligned} x_1' &= a(x_1^2 + x_2^2)x - b(x_1^2 + x_2^2)y \\ x_2' &= a(x_1^2 + x_2^2)y + b(x_1^2 + x_2^2)x \end{aligned}$$

with differentiable functions $a, b : \mathrm{R} \to \mathrm{R}$.

a) For the special case $a(z) = -z$ and $b(z) = 2$ find all fixedpoints and discuss their stability.

b) Transform the system to polar coordinates r, ϕ, with

$$r^2 = x_1^2 + x_2^2, \quad \phi = \arctan(\frac{x_1}{x_2}).$$

c) Does the transformation give any inside into the stability of fixedpoints?

Problem 3

We consider the system $x'(t) = Ax(t)$ with the fixed point $x^* = 0$.

a) Let $x^* = 0$ be asymptotically stable. Then there is a stepsize $\tau > 0$ such that the linear recursion

$$x_{k+1} = (I + \tau A)x_k, \quad k = 0, \dots, \tag{1}$$

is asymptotically stable.

b) Let all eigenvalues of A be complex (not real) and let $x^* = 0$ be stable and not asymptotically stable. Then the linear recursion is unstable for all $\tau > 0$.

c) Illustrate the result of b) in the special case

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

by computing explicit Euler approximations with a corresponding MATLAB programm for the initial value $x_\varepsilon = (\varepsilon, \varepsilon)^T$ with $\varepsilon = 10^{-2}, 10^4, 10^6$ and suitable final time T and stepsize $\tau > 0$. What happens, if the implicit Euler method is used?