Department of Mathematics & Computer Science
Freie Universität Berlin
Prof. Dr. Ralf Kornhuber, Maren-Wanda Wolf

Exercise 7 for the lecture

Numerics II

WS 2014/15

Due: till Tuesday, 9. December

Problem 1

A Butcher-scheme $\begin{array}{c|c} & \text{A} \\ \hline & b^T \end{array}$ with

$$A = \begin{pmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ \vdots & \vdots & \ddots & \\ a_{s1} & a_{s2} & \dots & a_{ss} \end{pmatrix}$$

defines a *diagonally implicit* Runge-Kutta (DIRK) method Ψ^τ of stage s. If $a_{11} = a_{22} = \ldots = a_{ss}$, Ψ^τ is called a *singly diagonally implicit* Runge-Kutta (SDIRK) method.

a) Describe the advantages of DIRK and SDIRK methods, over usual implicit Runge-Kutta methods.

b) Construct a 2-stage SDIRK method $\Psi^\tau_{2,3}$ of order $p = 3$.

c) Is $\Psi^\tau_{2,3}$ A-stable?

Problem 2

Let $E : \mathrm{R}^n \to \mathrm{R}$ be a convex functional and consider the associated *gradient flow*

$$x'(t) = -\nabla E(x(t)), \qquad x(0) = x_0, \tag{1}$$

where $\nabla E(x(t)) \in \mathrm{R}^n$ is the gradient of E at $x(t)$.

a) Show that $E(x(t)) \leq E(x_0)$ for all $t > 0$. Show then that even $E(x(t)) < E(x_0)$ if $\nabla E(x_0) \neq 0$.

b) Assume that E is strictly convex. Show that each fixed point of (1) is asymptotically stable.

c) Assume that E is strictly convex and coercive. Show that there exists a unique, asymptotically stable fixed point of (1).