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Exercise 1 for the lecture

Numerics II

WS 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumericsII.php

Due: Thu, 2015-11-05

Problem 1

Let $A \in \mathbb{R}^{d \times d}$. Show that the matrix exponential series

$$\exp(tA) = \sum_{k=0}^{\infty} \frac{1}{k!}(tA)^k$$

converges uniformly with respect to $t \in [0, T]$ for any time interval $[0, T]$.

Problem 2

Consider the differential equation

$$x'(t) = \lambda x(t) + \cos(t)e^{\lambda t}, \qquad t > 0 \tag{1}$$

with a real parameter λ and initial value $x(0) = x_0 \in \mathbb{R}$.

a) Rewrite equation (1) to an autonomous equation. (Hint: choose $y = (x, t)$).

b) Investigate existence and uniqueness of solutions with respect to λ. Do not use part c).

c) Find a closed representation of the solution x. How does $x(t)$ behave for $t \to \infty$ in dependence of λ and x_0?

d) Calculate the Wronski matrix and give the pointwise condition number of the differential equation (1).

Problem 3 (8 PP)

a) Implement a matlab programm `function [x, t] = RungeKuttaEx(f, x0, I, tau, b, A)`, which performs an explicit Runge-Kutta method, given by `b` and `A` from the Butcher scheme and the timestep `tau`, for the autonomous initial value problem

$$x'(t) = f(x(t)), \qquad t \in (\mathtt{I(1)}, \mathtt{I(2)}] \qquad x(\mathtt{I(1)}) = x_0$$

with right hand side $f : \mathbb{R}^d \to \mathbb{R}^d$ and initial value $x_0 \in \mathbb{R}^d$. The return value `x` should contain the solution and `t` the associated points in time as a vector.

b) Calculate the solution of the autonomous version of

$$x'(t) = \lambda x(t) + \cos(t)e^{\lambda t}, \qquad t > 0$$

on the intervall $[0, 10]$ with initial value $x_0 = 0$ using your program. Use Runge-Kutta-4, which is given by the Butcher scheme

$$\begin{array}{c|cccc} & 0 & & & \\ & 1/2 & 0 & & \\ & 0 & 1/2 & 0 & \\ & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 1/3 & 1/3 & 1/6. \end{array}$$

Plot your solution for $\lambda = -10, -100, -1000$ and timesteps $\tau = 0.001, 0.1$. Additionally, for each λ plot the discretization error over $\tau = 0.001 + k0.001$ for $k = 0, ..., 29$. Interprete your results.

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `graeser@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.