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Exercise 2 for the lecture

Numerics II

WS 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumericsII.php

Due: Thu, 2015-11-19

Problem 1

Let $B \in \mathbb{R}^{d\times d}$ and $\rho(B) = \max_{\lambda\in\sigma(B)} |\lambda|$ the spectral radius of B. Prove the following characterization of (asymptotic) stability of the linear recursion $x_{k+1} = Bx_k$.

a) If $\rho(B) \leq 1$ and every eigenvalue $\lambda \in \sigma(B)$ with $|\lambda| = 1$ fulfills $r(\lambda) = s(\lambda)$, i.e., algebraic and geometric multiplicity of λ coincide, then the recursion is stable.

b) If $\rho(B) < 1$, then the recursion is asymptotically stable.

Hint: Show that any Jordan block $J = \lambda I + N$ satisfies $\|J^k\| \leq |\lambda|^k p(k)$ for some polynomial p and use this to show the assertions.

Problem 2

Consider the one-dimensional odes

a) $x' = x - x^2$

b) $x' = -x + 4x^3 - x^5$

c) $x' = \begin{cases} 0, & x = 0 \\ -x^3 \sin(\frac{1}{x}), & x \neq 0. \end{cases}$

Find all fixed points and decide wether they are stable or unstable?

Please turn over.

Problem 3 (4 PP)
We consider the system $x'(t) = Ax(t)$ with the fixed point $x^* = 0$.

a) Let $x^* = 0$ be asymptotically stable. Then there is a stepsize $\tau > 0$ such that the linear recursion
$$x_{k+1} = (I + \tau A)x_k, \quad k = 0, \dots, \tag{1}$$
is asymptotically stable.

b) Let all eigenvalues of A be complex (not real) and let $x^* = 0$ be stable and not asymptotically stable. Then the linear recursion is unstable for all $\tau > 0$.

c) Illustrate the result of b) in the special case
$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
by computing explicit Euler approximations with a corresponding MATLAB programm for the initial value $x_\varepsilon = (\varepsilon, \varepsilon)^T$ with $\varepsilon = 10^{-2}, 10^4, 10^6$ and suitable final time T and stepsize $\tau > 0$. What happens, if the implicit Euler method is used?

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `graeser@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.