Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Carsten Gräser,

Exercise 3 for the lecture

Numerics II

WS 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumericsII.php

Due: Thu, 2015-11-26

Problem 1

a) Show that the stability function for the Runge-Kutta-4 method is given by

$$R(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}.$$

b) Show that the stability function for the implicit trapezoidal rule is given by

$$R(z) = \frac{1 + \frac{z}{2}}{1 - \frac{z}{2}}.$$

Problem 2

Let $\Phi^t = \exp(t)$. Show that if Ψ^τ is consistent with Φ^t with order p, then

$$\Psi^\tau = R(z) = \exp(z) + \mathcal{O}(z^{p+1}) \qquad \text{for } z \to 0$$

with $z = \lambda\tau$.

Problem 3

Consider the linear system

$$x'(t) = Ax(t). \tag{1}$$

Let $\Psi^\tau = R(\tau A)$ the discrete flow operator given by the rational function R of the matrix τA. Show that for all $\tau > 0$, Ψ^τ inherits (asymptotic) stability from (1) if R satifies the condition

$$\mathbb{C}_- \subset S = \{z \in \mathbb{C} | |R(z)| \leq 1\}.$$

Please turn over.

Problem 4

a) Compute the time step restriction for the Runge-Kutta-4 method applied to the linear system

$$x'(t) = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} x(t). \tag{2}$$

b) Sketch the stability domain for the method of Runge and visualize the time step restriction for (2).

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `graeser@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.