Fachbereich Mathematik & Informatik
Freie Universität Berlin
Prof. Dr. Carsten Gräser,

Exercise 7 for the lecture

Numerics II

WS 2015/2016

http://numerik.mi.fu-berlin.de/wiki/WS_2015/NumericsII.php

Due: Thu, 2016-01-21

Problem 1

a) Show that the parallel directional correction method associated with the Euclidean unit vectors e_i is the Jacobi method.

b) Show that the successive directional correction method associated with the Euclidean unit vectors e_i is the Gauß-Seidel method.

Problem 2

Derive from the cg method a method for *non-symmetric* A by application of A^T to $Ax = b$. Which Krylov spaces are spanned by this method? What can you say about the convergence properties?

Please turn over.

Problem 3 *6 PP*
Consider the linear system

$$AU = b \tag{1}$$

with the symmetric positive definite matrix $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$.

a) Implement the conjugate gradient method and the preconditioned conjugate gradient method as `matlab` functions

```
function [u, uk] = cg(A, b, u0, tol, uexact),
```

and

```
function [u, uk] = pcg(A, b, u0, tol, uexact, pre).
```

`u`, `uk`, `A`, `b`, `u0`, `tol`, and `uexact` denote the last iterate, a vector containing all iterates, the system matrix, the right hand side, the initial iterate, the error tolerance, and the exact solution, respectively. `pre` denotes a `function y = pre(x)` that applies the inverse $y = B^{-1}x$ of some preconditioner B. The iteration should stop if the energy norm $\|\cdot\|_A = \langle A\cdot, \cdot\rangle^{\frac{1}{2}}$ of the error is smaller than the tolerance.

b) Test your programs with the matrix of the model problem given in the lecture and the right hand side $b = AU$ where U is the pointwise evaluation of $(x_1 - x_1^2)(x_2 - x_2^2)$ for `u0` $= 0$, `tol` $= 10^{-8}$ and various choices of n. Use one Jacobi step and one symmetric Gauß-Seidel step, respectivly as preconditioner for the pcg-method. Plot the error over the number of iteration steps. Compare the results with the simple Jacobi and Gauß-Seidel method.

c) Augment your function `pcg` from a) with an error estimator. For Jacobi and symmetric Gauß-Seidel preconditioned cg method plot the estimated error

$$\|d\|_B, \qquad d = B^{-1}r_k$$

over the number of iteration steps and compare the results with the exact error from b).

General remarks

You have to do the exercises in groups of up 3 people. Be prepared to demonstrate your solutions to theoretical problems at the given date in the tutorial. Solutions for programming problems have to be submitted via e-mail to `graeser@mi.fu-berlin.de` with a subject starting by [NumericsII] and denoting all members of the group. Please follow the additional advise for programming exercises on the homepage.